HOMEWORK SUPPLEMENT

MATH 1593: Trigonometry

University of Central Oklahoma

Geometry Review

1. Specify the measure of the angle in degrees using the correct algebraic sign (+ or −):

   (a) $\frac{1}{3}$ rotation clockwise

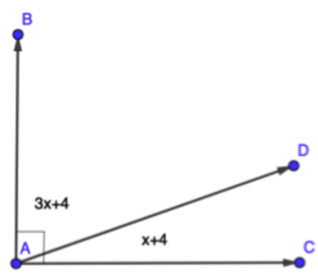
   (b) $\frac{7}{12}$ rotation counterclockwise

2. Find the measure of the angle:

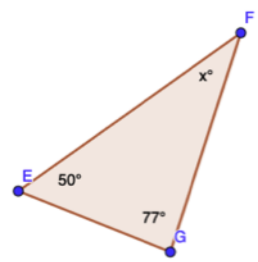
   (a) the complement of 39°

   (b) the supplement of 113°

3. Find the measures of two complementary angles with measures of $(3x + 4)°$ and $(x + 4)°$.



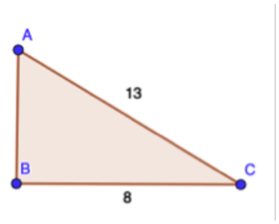
4. Find the measure of the missing angle for the given triangle.



5. Find the measures of two supplementary angles with measures of $(5x)°$ and $(7x)°$.

6. Let α, β, and γ be interior angles of a triangle. Find α, β, and γ if $\alpha = 2\beta$ and $\gamma = \alpha + \beta$.

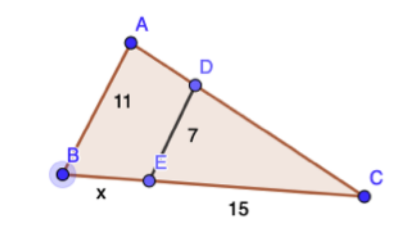
7. Find the measure of the missing side for the given right triangle.



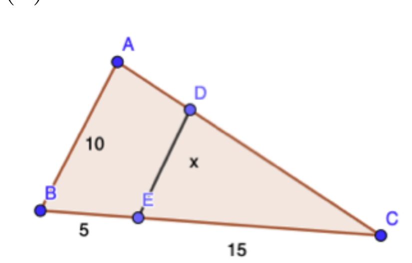
8. A 15-meter piece of string is stretched from the top of a 12-meter flagpole to the ground. How far is the base of the flagpole from the end of the piece of string?

9. Find the measure of x given that $\overline{AB} \parallel \overline{DE}$ in the following diagrams. Round to two decimal places.

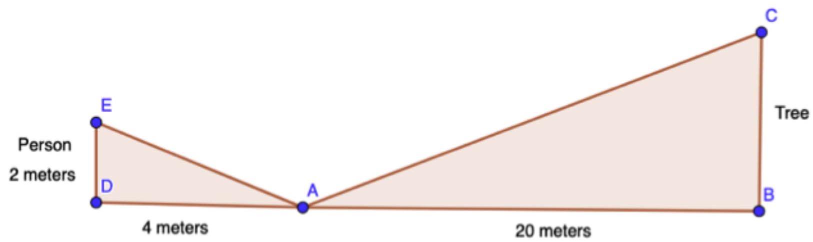
   (a)



   (b)



10. A person who wishes to find the height of a tree places a mirror horizontally on the ground 20 meters from the tree and finds that if they stand at point D which is 4 meters from the mirror A and their eye level is 2 meters, they can see the reflection of the tip of the tree. How high is the tree? Note that the angle of incidence of light to the mirror is equal to the angle of reflection



11. In a $45° - 45° - 90°$ triangle, if the length of the hypotenuse is 3 inches what is the length of each leg? Round answers to two decimal places.

12. In a $30° - 60° - 90°$ triangle, if the shortest leg is 3 inches, what is the length of the longest leg and the hypotenuse? Give exact answers.

Section 5.1: Angles

1. Draw an angle in standard position with the given measure.

   (a) $-80°$

   (b) $\frac{5\pi}{6}$

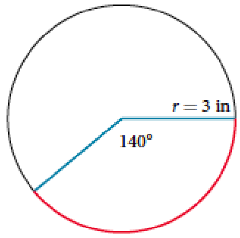
   (c) $\frac{\pi}{2}$

   (d) $415°$

   (e) $\frac{22\pi}{3}$

   (f) $-\frac{4\pi}{3}$

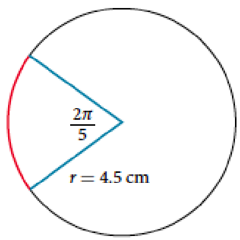
2. In a cirlcle with radius 3 inches, an arc is intercepted by a central angle with measure $140°$.



   (a) Find the arc length.

   (b) Find the area of the sector.

3. In a cirlcle with radius 4.5 centimeters, an arc is intercepted by a central angle with measure $\frac{2\pi}{5}$.



   (a) Find the arc length.

   (b) Find the area of the sector.

4. Convert the following angle measures from radians to degrees.

   (a) $\frac{3\pi}{4}$

   (b) $-\frac{5\pi}{12}$

   (c) $\frac{11\pi}{6}$

5. Convert the following angle measures from degrees to radians.

   (a) $-540°$

   (b) $-120°$

   (c) $150°$

6. Find the length of the arc of a circle of diameter 14 meters subtended by the central angle of $\frac{5\pi}{6}$.

7. Find the length of the arc of a circle of radius 10 centimeters subtended by the central angle of $50°$.

8. Find the area of the sector of a circle with diameter 10 feet and an angle of $\frac{\pi}{2}$ radians.

9. Find the area of the sector of a circle with central angle of $30°$ and a radius of 20 cm.

10. Find the angle between 0° and 360° that is coterminal to the given angle.

    (a) -110°

    (b) 700°

11. Find the angle between 0 and 2π in radians that is coterminal to the given angle.

    (a) $-\dfrac{\pi}{9}$

    (b) $\dfrac{44\pi}{9}$

12. A wheel of radius 14 inches is rotating 0.5 rad/s. What is the linear speed v, the angular speed in RPM (revolutions per minute), and the angular speed in deg/s? Round to two decimal places.

13. A person is standing on the equator of Earth (radius 3960 miles). What is his linear speed in miles per hour? What is his angular speed in radians per minute? Round to two decimal places.

14. Consider a clock with an hour hand and minute hand. What is the measure of the angle the minute hand traces in 20 minutes?

15. A sprinkler has a 25-foot spray and covers an angle of 60°. What is the area that the sprinkler covers? Round to one decimal place.

16. A windshield wiper that is 15 inches long including the blade and arm rotates 85°. If the rubber part is 12 inches long, what area does the wiper clear? Round to the nearest inch-squared.

17. Two cities have the same longitude. The latitude of city A is 9.00 degrees north and the latitude of city B is 30.00 degrees north. Assume the radius of the earth is 3960 miles. Find the distance between the two cities. Round to two decimal places.

18. You are flying on an airplane at 35,000 feet and see a city ahead at an angle of depression of 15°. How far is your aircraft's ground position from the city in miles, if one mile equals 5,280 ft?

Sections 5.2 and 5.3: Unit Circle: Sine, Cosine, and Other Trigonometric Functions

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1. Find the quadrant in which the terminal point determined by t lies.

   (a) $\sin(t) < 0$ and $\cos(t) < 0$
   (b) $\sin(t) > 0$ and $\cos(t) > 0$
   (c) $\sin(t) > 0$ and $\cos(t) < 0$
   (d) $\sin(t) < 0$ and $\cos(t) > 0$

2. Find the exact value of each trigonometric function.

   (a) $\sin\left(\frac{\pi}{2}\right)$
   (b) $\sin\left(\frac{\pi}{3}\right)$
   (c) $\sin\left(\frac{\pi}{4}\right)$
   (d) $\sin\left(\frac{\pi}{6}\right)$
   (e) $\sin(\pi)$
   (f) $\sin\left(\frac{3\pi}{2}\right)$
   (g) $\sin(0)$
   (h) $\cos\left(\frac{\pi}{6}\right)$
   (i) $\cos(0)$

3. Find the reference angle for the given angle.

   (a) $240°$
   (b) $\frac{5\pi}{4}$
   (c) $-\frac{11\pi}{3}$
   (d) $-\frac{\pi}{8}$
   (e) $-315°$

4. Find the reference angle, the quadrant of the terminal side, and sine and cosine of each angle. If the angle is not one of the angles on the unit circle, use a calculator and round to three decimal places.

   (a) $225°$
   (b) $300°$
   (c) $210°$
   (d) $250°$
   (e) $\frac{5\pi}{4}$
   (f) $\frac{7\pi}{6}$
   (g) $\frac{5\pi}{6}$
   (h) $\frac{7\pi}{4}$

5. If $\cos(t) = \frac{1}{7}$ and t is in the 4th quadrant, find $\sin(t)$.

6. If $\sin(t) = -\frac{1}{4}$ and t is in the 3rd quadrant, find $\cos(t)$.

7. Find the coordinates of the point on a circle with radius 15 corresponding to an angle of $220°$.

8. Simplify: $\sin\left(\frac{11\pi}{3}\right)\cos\left(-\frac{5\pi}{6}\right)$

9. Suppose a child enters a carousel that takes one minute to revolve once around. The child enters at the point (0,1), that is, on the due north position. Assume the carousel revolves counter clockwise. What are the coordinates of the child after 90 seconds?

10. Find the exact value of each trigonometric function.

(a) $\cot\left(\frac{\pi}{6}\right)$

(b) $\sec\left(\frac{\pi}{4}\right)$

(c) $\tan\left(\frac{\pi}{3}\right)$

(d) $\csc\left(\frac{\pi}{3}\right)$

11. Use reference angles to evaluate the expression.

(a) $\tan\left(\frac{5\pi}{6}\right)$

(b) $\sec\left(\frac{7\pi}{6}\right)$

(c) $\cot\left(\frac{13\pi}{6}\right)$

(d) $\tan\left(-\frac{7\pi}{4}\right)$

(e) $\csc\left(\frac{5\pi}{4}\right)$

(f) $\tan 225°$

(g) $\sec 300°$

(h) $\csc(-150°)$

(i) $\csc 210°$

12. If $\sin t = \frac{3}{4}$, and t is in quadrant II, find $\cos t$, $\sec t$, $\csc t$, $\tan t$, and $\cot t$.

13. If $\cos t = -\frac{1}{3}$, and t is in quadrant III, find $\sin t$, $\sec t$, $\csc t$, $\tan t$, and $\cot t$.

14. If $\tan t = \frac{12}{5}$, and $0 \leq t < \frac{\pi}{2}$, find $\sin t$, $\cos t$, $\sec t$, $\csc t$, and $\cot t$.

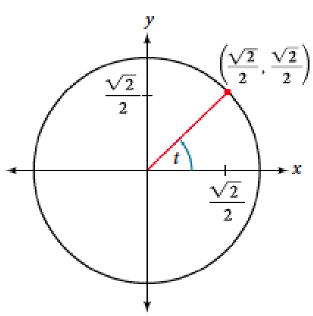
15. If $\sin t = \frac{\sqrt{3}}{2}$ and $\cos t = \frac{1}{2}$, find $\sec t$, $\csc t$, $\tan t$, and $\cot t$.

16. If $\cos t = \frac{1}{2}$, find $\cos(-t)$.

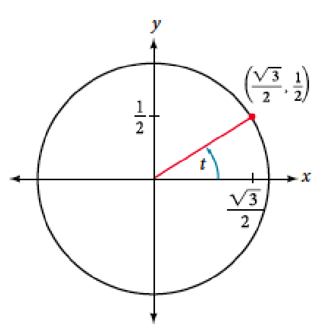
17. If $\tan t = -1.4$, find $\tan(-t)$.

18. Use the angle in the unit circle to find the value of each of the six trigonometric functions.

(a)



(b)



19. Use identities to rewrite the expression as a single trigonometric function.

(a) $\csc t \tan t$

(b) $\frac{\sec t}{\csc t}$

20. The terminal side of an angle θ passes through the point $(-4, 7)$. Calculate exactly the values of the six trigonometric functions of θ.

21. The terminal side of an angle θ passes through the point $\left(\frac{4}{5}, \frac{3}{5}\right)$. Calculate exactly the values of the six trigonometric functions of θ.

22. Use a fundamental identity to find the indicated function value. Rationalize denominators when necessary.

    (a) If $\sin\theta = -\frac{1}{2}$ and $\cos\theta = \frac{\sqrt{3}}{2}$, find $\tan\theta$.

    (b) If $\sin\theta = -\frac{4}{5}$ and $\cos\theta = \frac{3}{5}$, find $\cot\theta$.

23. Use a Pythagorean identity to find the indicated function value. Rationalize denominators when necessary.

    (a) If $\sin\theta = -\frac{1}{2}$, and the terminal side of θ lies in quadrant III, find $\cos\theta$.

    (b) If $\tan\theta = 4$, and the terminal side of θ lies in quadrant IV, find $\sec\theta$.

    (c) If $\cos\theta = -\frac{7}{15}$, and the terminal side of θ lies in quadrant III, find $\csc\theta$.

24. Use the appropriate identities to find the function value indicated. Rationalize denominators when necessary.

    (a) If $\tan\theta = -\frac{4}{3}$, and the terminal side of θ lies in quadrant II, find $\sin\theta$ and $\cos\theta$.

    (b) If $\tan\theta = 2$, and the terminal side of θ lies in quadrant III, find $\sin\theta$ and $\cos\theta$.

25. Show that $\sec\theta$ is an even function.

26. Show that $\tan\theta$ is an odd function.

Section 5.4: Right Triangle Trigonometry

1. Use the cofunction identities to fill in the blanks.

   (a) $\cos(34°) = \sin(\_\_\_\_°)$ (b) $\cos\left(\frac{\pi}{3}\right) = \sin(\_\_\_\_)$ (c) $\tan\left(\frac{\pi}{4}\right) = \cot(\_\_\_\_)$

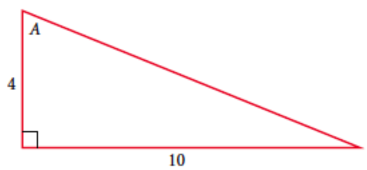
2. Find the lengths of the missing sides of a right triangle where side a is opposite angle A, side b is opposite angle B, and side c is the hypotenuse.

   (a) $\cos B = \frac{4}{5}$, $a = 10$

   (b) $\tan A = \frac{5}{12}$, $b = 6$

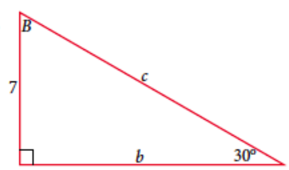
   (c) $\sin B = \frac{1}{\sqrt{3}}$, $a = 2$

3. Use the figure below to evaluate the six trigonometric functions of angle A.

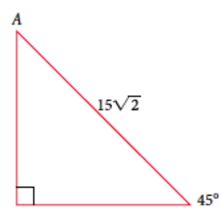


4. Solve for the unknown sides of the given triangles.

   (a)

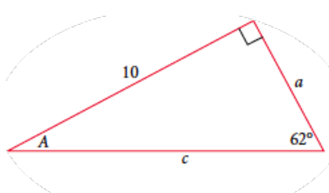


   (b)

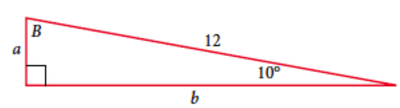


5. Use a calculator to find the length of each side to four decimal places.

   (a)

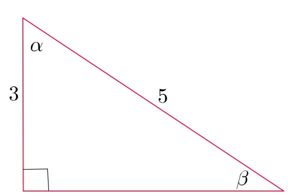


   (b)

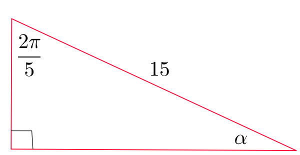


6. Solve for the unknown sides and angles.

   (a)



   (b)



7. If a plane takes off bearing N29°W and flies at 9 miles per hour for 2 hours and then makes a right turn (90°) and flies at 15 miles per hour for 3 hours, what is the bearing of the plane from its starting point? Round to the nearest whole number.
8. A radio tower is located 400 feet from a building. From a window in the building, a person determines that the angle of elevation to the top of the tower is 36°, and that the angle of depression to the bottom of the tower is 23°. How tall is the tower?
9. A 400-foot tall monument is located in the distance. From a window in the building, a person determines that the angle of elevation to the top of the monument is 18°, and that the angle of depression to the bottom of the monument is 3°. How far is the person from the monument?
10. There is a lightning rod on the top of a building. From a location 500 feet from the base of the building, the angle of elevation to the top of the building is measured to be 36°. From the same location, the angle of elevation to the top of the lightning rod is measured to be 38°. Find the height of the lightning rod.
11. A 23-ft ladder leans against a building so that the angle between the ground and the ladder is 80°. How high does the ladder reach up the side of the building?
12. Assuming that a 370-foot tall giant redwood grows vertically, if I walk a certain distance from the tree and measure the angle of elevation to the top of the tree to be 60°, how far from the base of the tree am I? Round your answer to the nearest foot.

Section 6.1: Graphs of Sine and Cosine Functions

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1. For the following functions, graph two full periods and state the amplitude, period, and midline. State the maximum and minimum y-values and their corresponding x-values on one period for $x \geq 0$. Then state the domain and range for each function. Give exact answers when possible. Round answers to two decimal places when necessary.

   (a) $f(x) = 2\sin x$

   (b) $f(x) = \dfrac{2}{3}\cos x$

   (c) $f(x) = \cos(2x)$

   (d) $f(x) = 4\cos(\pi x)$

2. For the following functions, graph one full period, starting at $x = 0$ and state the amplitude, period, and midline. State the maximum and minimum y-values and their corresponding x-values on one period for $x \geq 0$. State the phase shift and vertical translation, if applicable. State the domain and range for each function. Give exact answers when possible. Round answers to two decimal places when necessary.

   (a) $f(x) = 2\sin\left(x - \dfrac{5\pi}{6}\right)$

   (b) $f(x) = -\cos\left(x + \dfrac{\pi}{3}\right) + 1$

   (c) $f(x) = 4\cos\left(2\left(x + \dfrac{\pi}{4}\right)\right) - 3$

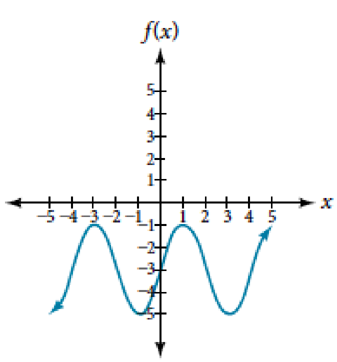
   (d) $f(x) = -\sin\left(\dfrac{1}{2}x + \dfrac{5\pi}{3}\right)$

   (e) $f(x) = 4\sin\left(\dfrac{\pi}{2}(x-3)\right) + 7$

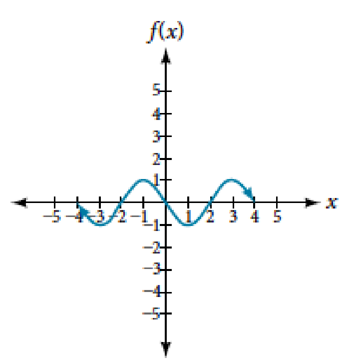
   (f) $y = 5\sin(5x + 20) - 2$

3. Determine the amplitude, midline, period, domain, range, and an equation involving the sine function for the graphs below.

   (a)

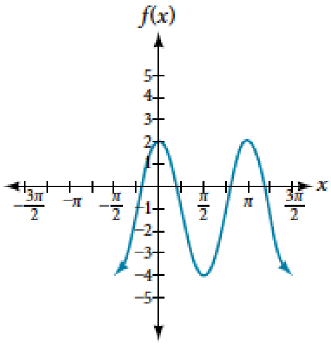


   (b)



4. Determine the amplitude, midline, period, domain, range, and an equation involving the cosine function for the graph below.

(a)



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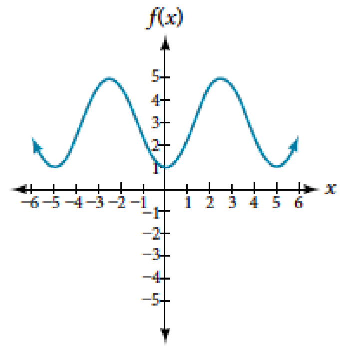

5. Let $f(x) = \sin x$

(a) On $[0, 2\pi)$, solve $f(x) = 0$.

(b) On $[0, 2\pi)$, solve $f(x) = \dfrac{1}{2}$.

(c) Evaluate $f\left(\dfrac{\pi}{2}\right)$

(d) On $[0, 2\pi)$, $f(x) = \dfrac{\sqrt{2}}{2}$. Find all values of x.

(e) On $[0, 2\pi)$, the maximum value(s) of the function occur(s) at what x-value(s)?

(f) On $[0, 2\pi)$, the minimum value(s) of the function occur(s) at what x-value(s)?

(g) Show that $f(-x) = -f(x)$. This means that $f(x) = \sin x$ is an odd function and possesses symmetry with respect to ____________.

6. For the following functions, graph the function then verbalize how the graph varies from the graph of $f(x) = \sin x$.

(a) $g(x) = x + \sin x$

(b) $h(x) = x \sin x$

7. A Ferris wheel is 25 meters in diameter and boarded from a platform that is 1 meter above the ground. The six o'clock position on the Ferris wheel is level with the loading platform. The wheel completes 1 full revolution in 10 minutes. The function $h(t)$ gives a person's height in meters above the ground t minutes after the wheel begins to turn.

(a) Find the amplitude, midline, and period of $h(t)$.

(b) Find a formula for the height function, $h(t)$.

(c) How high off the ground is a person after 5 minutes?

Section 6.2: Graphs of the Other Trigonometric Functions

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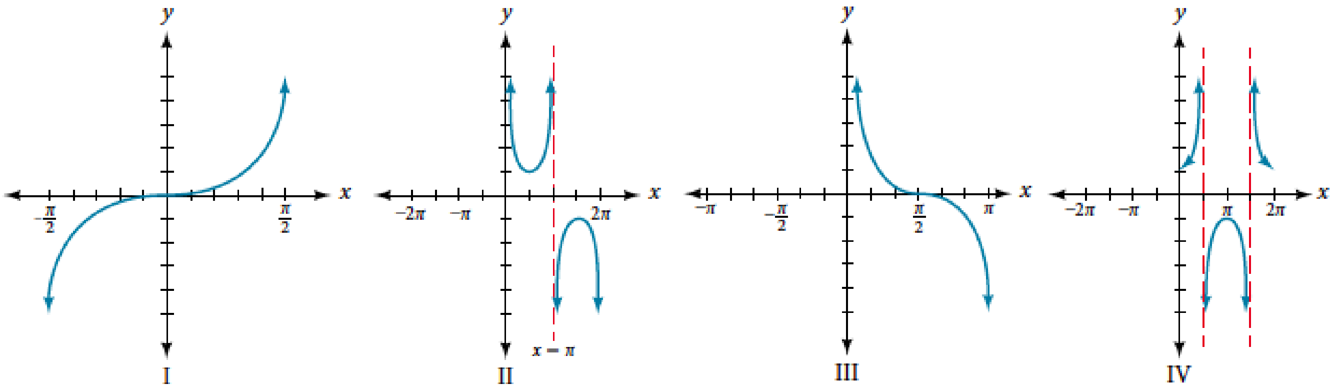
1. Match each trigonometric function to its graph graph.



   (a) $f(x) = \tan x$
   (b) $f(x) = \sec x$
   (c) $f(x) = \csc x$
   (d) $f(x) = \cot x$

2. State the period and horizontal shift of the following functions.

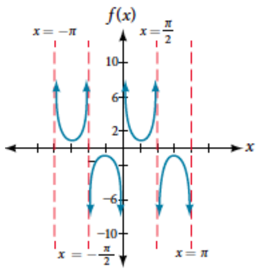
   (a) $f(x) = 2\tan(4x - 32)$
   (b) $f(x) = 2\sec\left(\frac{\pi}{4}(x+1)\right)$

3. Sketch two periods of the graph for each of the following functions. State the vertical stretch factor, period, asymptotes, domain, and range.

   (a) $f(x) = 2\sec\left(\frac{\pi}{4}(x+1)\right)$
   (b) $f(x) = 6\csc\left(\frac{\pi}{3}x + \pi\right)$
   (c) $f(x) = \tan\left(\frac{\pi}{2}x\right)$
   (d) $f(x) = \tan\left(x - \frac{\pi}{2}\right)$
   (e) $f(x) = \tan\left(x + \frac{\pi}{4}\right)$
   (f) $f(x) = 2\csc x$
   (g) $f(x) = 4\sec(3x)$
   (h) $f(x) = -3\cot(2x)$
   (i) $f(x) = \frac{9}{10}\csc(\pi x)$
   (j) $f(x) = 2\csc\left(x + \frac{\pi}{4}\right) - 1$

4. Find an equation for the graph of each function.

(a)



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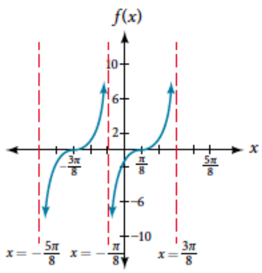

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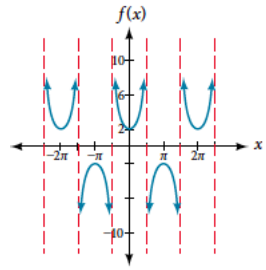

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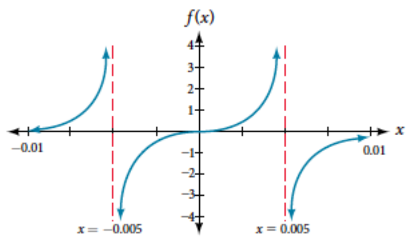

5. Use a graphing calculator to graph two periods of the given functions. Note: most graphing calculators do not have a cosecant button; therefore, you will need to input $\csc x$ as $\dfrac{1}{\sin x}$.

(a) $f(x) = |\csc x|$

(b) $f(x) = \dfrac{\csc x}{\sec x}$

6. The function $f(x) = 20\tan\left(\dfrac{\pi}{10}x\right)$ marks the distance in the movement of a light beam from a police car across a wall for time x, in seconds, and distance $f(x)$, in feet.

(a) Graph $f(x)$ on the interval $[0, 5]$.

(b) Find the stretching factor, period, and asymptote.

(c) Evaluate $f(1)$ and $f(2.5)$ and interpret the function's values at those inputs.

Section 6.3: Inverse Trigonometric Functions

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1. Why must the domain of the sine function, $\sin x$, be restricted to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ for the inverse sine function to exist?

2. Evaluate the following expressions exactly. Be sure to put your answers in terms of radians.

   (a) $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$

   (b) $\sin^{-1}\left(-\frac{1}{2}\right)$

   (c) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

   (d) $\tan^{-1}(1)$

   (e) $\tan^{-1}(\sqrt{3})$

   (f) $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

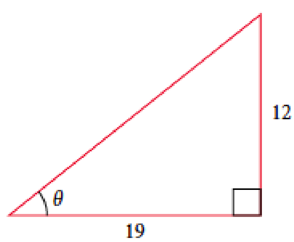
3. Use a calculator to evaluate each expression. Give answers in radiants. Round answers to the nearest hundredth where necessary.

   (a) $\cos^{-1}(-0.4)$

   (b) $\arccos\left(\frac{3}{5}\right)$

   (c) $\tan^{-1}(6)$

4. Find the angle, θ, (in radians) for the right triangle given below. Round answers to the nearest hundredth.



5. Find the exact value, if possible, without a calculator. If it is not possible, explain why.

   (a) $\sin^{-1}(\cos(\pi))$

   (b) $\tan^{-1}\left(\sin\left(\frac{\pi}{3}\right)\right)$

   (c) $\sin^{-1}\left(\sin\left(\frac{5\pi}{6}\right)\right)$

   (d) $\cos\left(\sin^{-1}\left(\frac{4}{5}\right)\right)$

   (e) $\sin\left(\cos^{-1}\left(\frac{3}{5}\right)\right)$

   (f) $\cos\left(\tan^{-1}\left(\frac{12}{5}\right)\right)$

6. Find the exact value of the expression in terms of x with the help of a reference triangle.

   (a) $\tan(\sin^{-1}(x-1))$

   (b) $\cos\left(\sin^{-1}\left(\frac{1}{x}\right)\right)$

7. Simplify $\cos\left(\sin^{-1}\left(\frac{x}{x+1}\right)\right)$.

8. Graph $y = \arccos x$ and state the domain and range of the function.

9. Suppose a 13-foot ladder is leaning against a building, reaching to the bottom of a second-floor window 12 feet above the ground. What angle, in radians, does the ladder make with the building? Round to the nearest 2 decimal places.

10. An isosceles triangle has two congruent sides of length 9 inches. The remaining side has a length of 8 inches. Find the angle, in degrees, that a side of 9 inches makes with the 8-inch side. Round to the nearest decimal place.

11. The line $y = -\frac{3}{7}x$ passes through the origin in the xy-plane. What is the measure of the angle, in radians, that the line makes with the negative x-axis?

Section 7.1: Solving Trigonometric Equations with Identities

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1. Use the fundamental identities to fully simplify each expression.

   (a) $\sin x \cos x \sec x$

   (b) $\tan x \sin x + \sec x \cos^2 x$

   (c) $\dfrac{\cot t + \tan t}{\sec(-t)}$

   (d) $\dfrac{1 + \tan^2 \theta}{\csc^2 \theta} + \sin^2 \theta + \dfrac{1}{\sec^2 \theta}$

   (e) $\dfrac{1 - \cos^2 x}{\tan^2 x} + 2\sin^2 x$

2. Simplify $\dfrac{\tan x + \cot x}{\csc x}$ by rewriting in terms of $\cos x$.

3. Simplify $\dfrac{1 - \sin x}{1 + \sin x} - \dfrac{1 + \sin x}{1 - \sin x}$ by rewriting in terms of $\sec x$ and $\tan x$.

4. Verify the identity.

   (a) $\cos x - \cos^3 x = \cos x \sin^2 x$

   (b) $(\sin x + \cos x)^2 = 1 + 2\sin x \cos x$

   (c) $\cos^2 x - \tan^2 x = 2 - \sin^2 x - \sec^2 x$

Section 7.2: Sum and Difference Identities

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1. Evaluate exactly.

   (a) $\cos\left(\frac{7\pi}{12}\right)$

   (b) $\sin\left(\frac{11\pi}{12}\right)$

   (c) $\sin(-75^\circ)$

   (d) $\tan\left(-\frac{\pi}{12}\right)$

   (e) $\tan(105^\circ)$

   (f) $\cos(435^\circ)$

2. Rewrite in terms of $\sin x$ and $\cos x$ using the sum and difference identities.

   (a) $\sin\left(x - \frac{3\pi}{4}\right)$

   (b) $\cos\left(x + \frac{2\pi}{3}\right)$

3. Simplify the given expression.

   (a) $\sec\left(\frac{\pi}{2} - \theta\right)$

   (b) $\tan\left(\frac{\pi}{2} - \theta\right)$

   (c) $\sin(2x)\cos(5x) - \sin(5x)\cos(2x)$

   (d) $\dfrac{\tan\left(\frac{3}{2}x\right) - \tan\left(\frac{7}{5}x\right)}{1 + \tan\left(\frac{3}{2}x\right)\tan\left(\frac{7}{5}x\right)}$

4. Given that $\sin a = \frac{2}{3}$ and $\cos b = -\frac{1}{4}$, with a and b both in the interval $\left[\frac{\pi}{2}, \pi\right)$, find $\sin(a+b)$ and $\cos(a-b)$.

5. Given that $\sin a = \frac{4}{5}$ and $\cos b = \frac{1}{3}$, with a and b both in the interval $\left[0, \frac{\pi}{2}\right)$, find $\sin(a-b)$ and $\cos(a+b)$.

6. Evaluate exactly.

   (a) $\sin\left(\cos^{-1}(0) - \cos^{-1}\left(\frac{1}{2}\right)\right)$

   (b) $\cos\left(\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) + \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)$

   (c) $\tan\left(\sin^{-1}\left(\frac{1}{2}\right) - \cos^{-1}\left(\frac{1}{2}\right)\right)$

7. Verify the identity.

   (a) $\tan\left(x + \frac{\pi}{4}\right) = \dfrac{\tan x + 1}{1 - \tan x}$

   (b) $\dfrac{\cos(a+b)}{\cos a \cos b} = 1 - \tan a \tan b$

Section 7.3: Double-Angle, Half-Angle, and Reduction Formulas

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1. Explain how to determine the double-angle formula for $\tan(2x)$ using the double-angle formulas for $\cos(2x)$ and $\sin(2x)$.

2. Find the exact values of $\sin(2x)$, $\cos(2x)$, and $\tan(2x)$ without solving for x.

   (a) If $\sin x = \frac{1}{8}$, and x is in quadrant I.

   (b) If $\cos x = -\frac{1}{2}$, and x is in quadrant III.

   (c) If $\tan x = -8$, and x is in quadrant IV.

3. If $\cos(2\theta) = \frac{3}{5}$ and $90° \leq \theta \leq 180°$, find the values of $\sin\theta$, $\cos\theta$, and $\tan\theta$.

4. Given $\cos(b) = \frac{2}{3}$, with b in quadrant II, find $\tan(2b)$.

5. Simplify to one trigonometric expression.

   (a) $2\sin\left(\frac{\pi}{4}\right) 2\cos\left(\frac{\pi}{4}\right)$

   (b) $4\sin\left(\frac{\pi}{8}\right)\cos\left(\frac{\pi}{8}\right)$

6. Evaluate exactly using half-angle formulas.

   (a) $\sin\left(\frac{\pi}{8}\right)$

   (b) $\cos\left(-\frac{11\pi}{12}\right)$

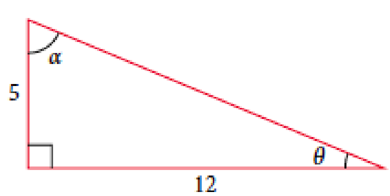
7. Find the exact values of $\sin\left(\frac{x}{2}\right)$, $\cos\left(\frac{x}{2}\right)$, $\tan\left(\frac{x}{2}\right)$ without solving for x, when $0° \leq x \leq 360°$.

   (a) If $\tan x = -\frac{4}{3}$, and x is in quadrant IV.

   (b) If $\sin x = -\frac{12}{13}$, and x is in quadrant III.

   (c) If $\sec x = -4$, and x is in quadrant II.

8. Use the figure below to find the requested half and double angles.



   (a) Find $\sin(2\alpha)$, $\cos(2\alpha)$, $\tan(2\alpha)$.

   (b) Find $\sin\left(\frac{\alpha}{2}\right)$, $\cos\left(\frac{\alpha}{2}\right)$, $\tan\left(\frac{\alpha}{2}\right)$.

9. Simplify each expression. Do not evaluate.

   (a) $\cos^2(28°) - \sin^2(28°)$

   (b) $1 - 2\sin^2(17°)$

   (c) $\cos^2(9x) - \sin^2(9x)$

   (d) $6\sin(5x)\cos(5x)$

10. Verify the identity.

(a) $(\sin t - \cos t)^2 = 1 - \sin(2t)$

(b) $\dfrac{\sin(2\theta)}{1 + \cos(2\theta)} \tan^2\theta = \tan^3\theta$

(c) $\sin(2x) = \dfrac{2\tan x}{1 + \tan^2 x}$

(d) $(\sin^2 x - 1)^2 = \cos(2x) + \sin^4 x$

Section 7.4: Sum-to-Product and Product-to-Sum Formulas

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1. Rewrite the product as a sum or difference.

   (a) $16\sin(16x)\sin(11x)$ (b) $20\cos(36t)\cos(6t)$ (c) $\sin(3x)\cos(5x)$

2. Rewrite the sum or difference as a product.

   (a) $\cos(6t)+\cos(4t)$ (b) $\cos(7x)+\cos(-7x)$ (c) $\sin h-\sin(3h)$

3. Evaluate the product using a sum or difference of two functions. Then evaluate exactly.

   (a) $\cos(45°)\cos(15°)$ (b) $\cos(45°)\sin(15°)$ (c) $\sin(-45°)\sin(-15°)$

Section 7.5: Solving Trigonometric Equations

---

1. Will there always be solutions to trigonometric equations? If not, describe an equation that would not have a solution. Explain why or why not.

2. Find all possible exact solutions.

   (a) $2\sin(2\theta) = \sqrt{3}$

   (b) $2\cos(2\theta) = -\sqrt{2}$

3. Solve exactly on $[0, 2\pi)$.

   (a) $\tan x = 1$

   (b) $4\sin^2 x - 2 = 0$

   (c) $\csc^2 x - 4 = 0$

   (d) $2\sin(2\theta) = \sqrt{3}$

   (e) $\cos(2\theta) = -\dfrac{\sqrt{3}}{2}$

   (f) $2\cos^2 t + \cos t = 1$

   (g) $\cos^2 \theta = \dfrac{1}{2}$

4. Solve exactly on $[0, 2\pi)$ by first FACTORING the equation.

   (a) $8\sin^2 x + 6\sin x + 1 = 0$

   (b) $\sin^2 x + \sin x - 2 = 0$

   (c) $\sec^2 x + \sec x = 0$

   (d) $4\sin^2 x = 3 + 4\sin x$

5. Solve exactly on $[0, 2\pi)$ by using the QUADRATIC formula.

   (a) $3\cos^2 x - 2\cos x - 2 = 0$

   (b) $-\tan^2 x - \tan x = 2$

6. Solve exactly on $[0, 2\pi)$ using any method.

   (a) $\tan^2 x + 5\tan x = 1$

   (b) $\sin^2 x + 2\sin x + 1 = 0$

   (c) $\tan^2 x - \sqrt{3}\tan x = 0$

   (d) $\sin^2 x - 2\sin x - 4 = 0$

   (e) $2\cos^2 x = \cos x$

7. Solve exactly on $[0, 2\pi)$. Look for opportunities to use trigonometric identities.

   (a) $\sin^2 x - \cos^2 x - \sin x = 0$

   (b) $\sin^2 x + \cos^2 x = 0$

   (c) $\sin(2x) - \sin x = 0$

   (d) $\cos(2x) - \cos x = 0$

   (e) $\sec^2 x = 2$

   (f) $10\sin x\cos x = 6\cos x$

   (g) $-3\sin t = 15\cos t\sin t$

   (h) $\tan x = 3\sin x$

8. Solve the equations algebraically, and then use a calculator to find the values on the interval $[0, 2\pi)$. Round to two decimal places.

   (a) $\tan^2 x + 3\tan x - 3 = 0$

   (b) $\sin^2 x - 2\cos^2 x = 0$

   (c) $4\cos^2 x + 5\cos x - 6 = 0$

Section 8.1: Non-right Triangles: Law of Sines

1. Assume α is opposite side a, β is opposite side b, and γ is opposite side c. Solve each triangle, if possible. Round each answer to the nearest tenth.

   (a) $\alpha = 35°$, $\gamma = 73°$, $c = 20$

   (b) $\alpha = 60°$, $\beta = 60°$, $\gamma = 60°$

   (c) $\beta = 95°$, $\gamma = 30°$, $b = 10$

2. Assume α is opposite side a, β is opposite side b, and γ is opposite side c. Use the Law of Sines to find side b when $\alpha = 37°$, $\beta = 49°$, and $c = 5$. Round each answer to the nearest hundredth.

3. Assume α is opposite side a, β is opposite side b, and γ is opposite side c. Determine whether there is no triangle, one triangle, or two triangles. Then solve each triangle, if possible. Round each answer to the nearest tenth.

   (a) $\gamma = 113°$, $b = 10$, $c = 32$

   (b) $\gamma = 80°$, $b = 3.5$, $c = 5.3$

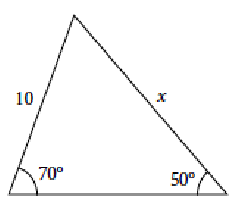
   (c) $\alpha = 35°$, $a = 12$, $c = 17$

   (d) $\alpha = 43°$, $a = 7$, $c = 9$

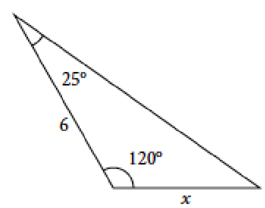
   (e) $\beta = 119°$, $a = 11.3$, $b = 8.2$

4. Find the length of side x. Round to the nearest tenth.

   (a)

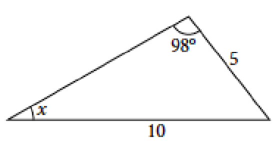


   (b)

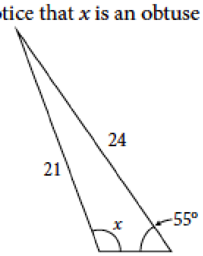


5. Find the measure of angle x, if possible. Round to the nearest tenth.

   (a)

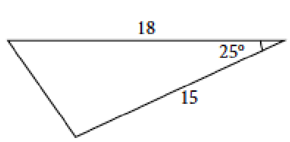


   (b) Notice that x is an obtuse angle.

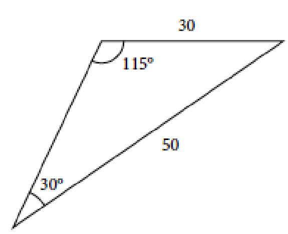


6. Find the area of each triangle. Round each answer to the nearest tenth.

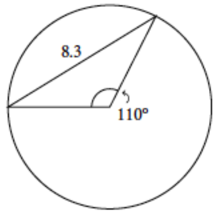
   (a)



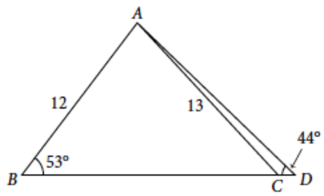
   (b)



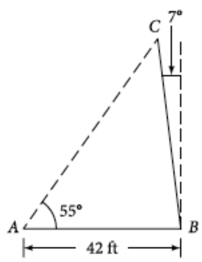
7. Find the diameter of the circle in the figure below. Round to the nearest tenth.



8. Find AD in the figure below. Round to the nearest tenth.



9. A pole leans away from the sun at an angle of 7° to the vertical, as shown below. When the elevation of the sun is 55°, the pole casts a shadow 42 feet long on the level ground. How long is the pole? Round answer to the nearest tenth.



10. In order to estimate the height of a building, two students stand at a certain distance from the building at street level. From this point, they find the angle of elevation from the street to the top of the building to be 35°. They then move 250 feet closer to the building and find the angle of elevation to be 53°. Assuming that the street is level, estimate the height of the building to the nearest foot.

11. A man and a woman standing 3.5 miles apart, facing each other, simultaneously spot a hot air balloon between them. If the angle of elevation from the man to the balloon is 27°, and the angle of elevation from the woman to the balloon is 41°, find the altitude of the balloon to the nearest foot.

12. Two streets meet at an 80° angle. At the corner, a park is being built in the shape of a triangle. Find the area of the park if, along one road, the park measures 180 feet, and along the other road, the park measures 215 feet.

Section 8.2: Non-right Triangles: Law of Cosines

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1. Explain the relationship between the Pythagorean Theorem and the Law of Cosines.
2. Assume α is opposite side a, β is opposite side b, and γ is opposite side c. If possible, solve each triangle for the unknown side. Round to the nearest tenth.

   (a) $\gamma = 41.2°$, $a = 2.49$, $b = 3.13$ (b) $\gamma = 115°$, $a = 18$, $b = 23$

3. Use the Law of Cosines to solve for the missing angle of the oblique triangle. Round to the nearest tenth.

   (a) $a = 42$, $b = 19$, $c = 30$; Find angle α. (b) $a = 13$, $b = 22$, $c = 28$; Find angle α.

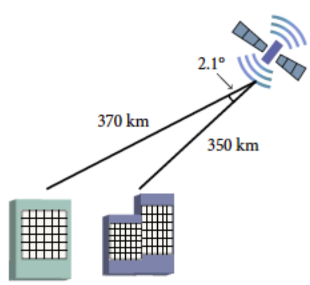
4. Solve the triangle. Round to the nearest tenth.

   (a) $\alpha = 35°$, $b = 8$, $c = 11$ (b) $a = 13$, $b = 11$, $c = 15$

5. Use Heron's formula to find the AREA of the triangle. Round to the nearest two decimal places.

   (a) Find the area of a triangle with sides of length 18 in, 21 in, and 32 in.
   (b) $a = 12.4$ ft, $b = 13.7$ ft, $c = 20.2$ ft

6. The sides of a parallelogram are 11 feet and 17 feet. The longer diagonal is 22 feet. Find the length of the shorter diagonal.
7. A satellite calculates the distances and angle show in the figure below. Find the distance between the two cities. Round answers to the nearest tenth.



8. A pilot flies in a straight path for 1 hour 30 min. She then makes a course correction, heading 10° to the right of her original course, and flies 2 hours in the new direction. If she maintains a constant speed of 680 miles per hour, how far is she from her starting position?
9. Two airplanes take off in different directions. One travels 300 mph due west and the other travels 25° north of west at 420 mph. After 90 minutes, how far apart are they, assuming they are flying at the same altitude?
10. Find the area of a triangular piece of land that measures 30 feet on one side and 42 feet on another; the included angle measures 132°. Round to the nearest whole square foot.

Section 8.3: Polar Coordinates

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1. Explain why the points $\left(-3, \frac{\pi}{2}\right)$ and $\left(3, -\frac{\pi}{2}\right)$ are the same.

2. Covert the given polar coordinates to Cartesian coordinates. Remember to consider the quadrant in which the given point is located when determining θ for the point.

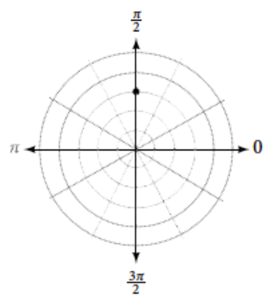
   (a) $\left(7, \frac{7\pi}{6}\right)$ (b) $(5, \pi)$ (c) $\left(-3, \frac{\pi}{6}\right)$

3. Convert the given Cartesian coordinates to polar coordinates with $r > 0$, $0 \le \theta < 2\pi$. Round your answer to three decimal places. Remember to consider the quadrant in which the given point is located.

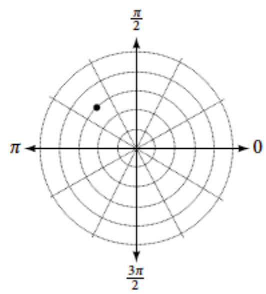
   (a) $(4, 2)$ (b) $(3, -5)$ (c) $(-10, -13)$

4. Find the polar coordinations of the point.

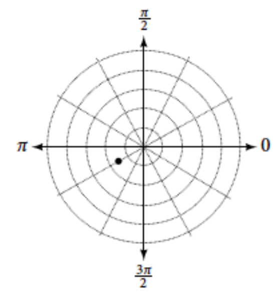
   (a)



   (b)



   (c)



5. Plot the following points.

   (a) $\left(-2, \frac{\pi}{3}\right)$ (b) $\left(-1, -\frac{\pi}{2}\right)$ (c) $\left(5, \frac{\pi}{2}\right)$ (d) $\left(-2, \frac{\pi}{4}\right)$

6. Convert the equation from rectangular to polar form and graph in polar coordinates.

   (a) $2x + 7y = -3$ (b) $(x+2)^2 + (y+3)^2 = 13$ (c) $x^2 + y^2 = 3x$

7. Convert the equation from polar to rectangular form and graph on the rectangular plane.

   (a) $r = -4$ (b) $\theta = \frac{\pi}{4}$ (c) $r = -10\sin\theta$

Section 8.5: Polar Form of Complex Numbers

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1. Find the absolute value of the given complex number.

   (a) $-7+i$ (b) $\sqrt{2}-6i$

2. Write the complex number in polar form.

   (a) $4-4\sqrt{3}i$ (b) $-\frac{1}{2}-\frac{1}{2}i$ (c) $\sqrt{3}+i$

3. Convert the complex number from polar to rectangular form.

   (a) $z=7\left(\cos\left(\frac{\pi}{6}\right)+i\sin\left(\frac{\pi}{6}\right)\right)$
   (b) $z=2\left(\cos\left(\frac{\pi}{3}\right)+i\sin\left(\frac{\pi}{3}\right)\right)$
   (c) $z=3\left(\cos\left(240^\circ\right)+i\sin\left(240^\circ\right)\right)$

4. Find z_1z_2 in polar form. That is, find the product of z_1 and z_2.

   (a) $z_1=2\sqrt{3}\left(\cos\left(116^\circ\right)+i\sin\left(116^\circ\right)\right)$; $z_2=2\left(\cos\left(82^\circ\right)+i\sin\left(82^\circ\right)\right)$
   (b) $z_1=3\left(\cos\left(120^\circ\right)+i\sin\left(120^\circ\right)\right)$; $z_2=\frac{1}{4}\left(\cos\left(60^\circ\right)+i\sin\left(60^\circ\right)\right)$
   (c) $z_1=4\left(\cos\left(\frac{\pi}{2}\right)+i\sin\left(\frac{\pi}{2}\right)\right)$; $z_2=2\left(\cos\left(\frac{\pi}{4}\right)+i\sin\left(\frac{\pi}{4}\right)\right)$

5. Find $\frac{z_1}{z_2}$ in polar form.

   (a) $z_1=21\left(\cos\left(135^\circ\right)+i\sin\left(135^\circ\right)\right)$; $z_2=3\left(\cos\left(65^\circ\right)+i\sin\left(65^\circ\right)\right)$
   (b) $z_1=5\sqrt{2}\left(\cos\left(\pi\right)+i\sin\left(\pi\right)\right)$; $z_2=\sqrt{2}\left(\cos\left(\frac{2\pi}{3}\right)+i\sin\left(\frac{2\pi}{3}\right)\right)$

6. Evaluate each expression using De Moivre's Theorem. Write your answer in rectangular form.

   (a) $(-1+i)^5$ (c) $(4-4i)^8$
   (b) $(1-\sqrt{3}i)^4$ (d) $(4\sqrt{3}+4i)^7$

7. Find the cube roots of the following complex numbers in polar form.

   (a) $-\frac{27}{2}-\frac{27\sqrt{3}}{2}i$ (b) $-16+16\sqrt{3}i$ (c) $4\sqrt{2}-4\sqrt{2}i$

Section 8.8: Vectors

1. Given a vector with initial point $(-4, 2)$ and terminal point $(3, -3)$, find an equivalent vector whose initial point is $(0, 0)$. Write the vector in component form $\langle a, b\rangle$.

2. Determine whether the two vectors $\vec{u}$ and $\vec{v}$ are equal, where $\vec{u}$ has an initial point P_1 and a terminal point P_2 and $\vec{v}$ has an initial point P_3 and a terminal point P_4.

   (a) $P_1 = (5, 1)$, $P_2 = (3, -2)$, $P_3 = (-1, 3)$, and $P_4 = (9, -4)$

   (b) $P_1 = (3, 7)$, $P_2 = (2, 1)$, $P_3 = (1, 2)$, and $P_4 = (-1, -9)$

3. Given initial point $P_1 = (6, 0)$ and terminal point $P_2 = (-1, -3)$, write the vector $\vec{v}$ in terms of $\vec{i}$ and $\vec{j}$.

4. Let $\vec{u} = \vec{i} + 5\vec{j}$, $\vec{v} = -2\vec{i} - 3\vec{j}$, and $\vec{w} = 4\vec{i} - \vec{j}$. Find the following.

   (a) $\vec{u} + (\vec{v} - \vec{w})$ (b) $4\vec{v} + 2\vec{u}$

5. Let $\vec{u} = \langle -3, 4\rangle$ and $\vec{v} = \langle -2, 1\rangle$. Find the following.

   (a) $\vec{u} + \vec{v}$ (b) $\vec{u} - \vec{v}$ (c) $2\vec{u} - 3\vec{v}$

6. Let $\vec{v} = -4\vec{i} + 3\vec{j}$. Find a vector that is half the length and points in the same direction as $\vec{v}$.

7. Find a unit vector in the same direction as the given vector.

   (a) $\vec{v} = 3\vec{i} + 4\vec{j}$ (b) $\vec{u} = -14\vec{i} + 2\vec{j}$

8. Find the magnitude and direction of the vector, $0 \le \theta < 2\pi$.

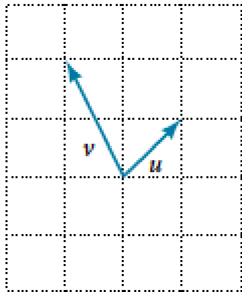
   (a) $\langle 6, 5\rangle$ (b) $\langle 2, -5\rangle$

9. Given $\vec{u} = -\vec{i} - \vec{j}$ and $\vec{v} = \vec{i} + 5\vec{j}$, calculate $\vec{u} \cdot \vec{v}$.

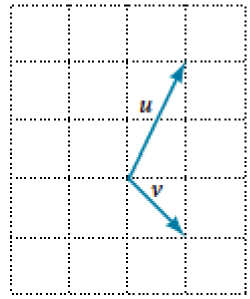
10. Let $\vec{v} = \langle -1, 4\rangle$. Sketch $\vec{v}$, $3\vec{v}$, and $\dfrac{1}{2}\vec{v}$.

11. Use the vectors shown to sketch $\vec{u} + \vec{v}$, $\vec{u} - \vec{v}$, and $2\vec{u}$.

   (a)



   (b)



12. A 60-pound box is resting on a ramp that is inclined 12°. Rounding to the nearest tenth,

    (a) Find the magnitude of the normal (perpendicular) component of the force.

    (b) Find the magnitude of the component of the force that is parallel to the ramp.

13. Find the magnitude of the horizontal and vertical components of a vector with magnitude 5 pounds pointed in a direction of 55° above the horizontal. Round to the nearest hundredth.

14. A man starts walking from home and walks 3 miles at 20° north of west, then 5 miles at 10° west of south, then 4 miles at 15° north of east. If he walked straight home, how far would he have to walk, and in what direction?

15. An airplane needs to head due north, but there is a wind blowing from 45° west of south at 60 km/hr. The plane flies with an airspeed of 550 km/hr. To end up flying due north, how many degrees west of north will the pilot need to fly the plane?

Selected Answers

Geometry Review

1a) $-120°$

2a) $51°$

2b) $67°$

3) $65.5°$, $24.5°$

4) $53°$

5) $75°$, $105°$

6) $\alpha = 60°$, $\beta = 30°$, $\gamma = 90°$

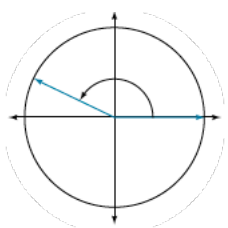
8) 9 meters

9b) 7.5

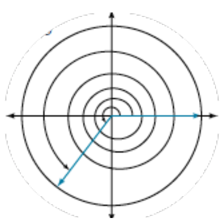
12) hypotenuse: 6 in; longest leg: $3\sqrt{3}$ in

Section 5.1

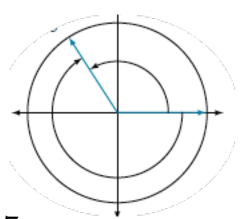
1b)



1e)



1f)



2b) $\dfrac{7\pi}{2} \approx 11 \text{ in}^2$

3b) $\dfrac{81\pi}{20} \approx 12.72 \text{ cm}^2$

4b) $-75°$

5a) -3π

5c) $\dfrac{5\pi}{6}$

7) $\dfrac{25\pi}{9} \approx 8.73$ cm

9) $\dfrac{100\pi}{3} \text{ cm}^2$

10a) $250°$

11b) $\dfrac{8\pi}{9}$

12) 7 in/s, 4.77 RPM, 28.65 deg/s

14) $120°$

Sections 5.2 and 5.3

1b) I

1d) IV

2b) $\dfrac{\sqrt{3}}{2}$

2e) 0

2g) 0

3a) $60°$

3c) $\dfrac{\pi}{3}$

3d) $\dfrac{\pi}{8}$

4b) $60°$, Quadrant IV, $\sin(300°) = -\dfrac{\sqrt{3}}{2}$, $\cos(300°) = \dfrac{1}{2}$

4f) $\dfrac{\pi}{6}$, Quadrant III, $\sin\left(\dfrac{7\pi}{6}\right) = -\dfrac{1}{2}$, $\cos\left(\dfrac{7\pi}{6}\right) = -\dfrac{\sqrt{3}}{2}$

4h) $\dfrac{\pi}{4}$, Quadrant IV, $\sin\left(\dfrac{7\pi}{4}\right) = -\dfrac{\sqrt{2}}{2}$, $\cos\left(\dfrac{7\pi}{4}\right) = \dfrac{\sqrt{2}}{2}$

6) $-\dfrac{\sqrt{15}}{4}$

9) $(0, -1)$

10a) $\sqrt{3}$

10b) $\sqrt{2}$

11b) $-\dfrac{2\sqrt{3}}{3}$

11c) $\sqrt{3}$

11g) 2

13) $\sin t = -\dfrac{2\sqrt{2}}{3}$, $\sec t = -3$, $\csc t = -\dfrac{3\sqrt{2}}{4}$, $\tan t = 2\sqrt{2}$, $\cot t = \dfrac{\sqrt{2}}{4}$

15) $\sec t = 2$, $\csc t = \dfrac{2\sqrt{3}}{3}$, $\tan t = \sqrt{3}$, $\cot t = \dfrac{\sqrt{3}}{3}$

17) 1.4

18a) $\sin t = \dfrac{\sqrt{2}}{2}$, $\cos t = \dfrac{\sqrt{2}}{2}$, $\tan t = 1$, $\cot t = 1$, $\sec t = \sqrt{2}$, $\csc t = \sqrt{2}$

19b) $\tan t$

Section 5.4

1b) $\dfrac{\pi}{6}$

1c) $\dfrac{\pi}{4}$

2a) $b = \dfrac{15}{2}$, $c = \dfrac{25}{2}$

2b) $a = \dfrac{5}{2}$, $c = \dfrac{13}{2}$

2c) $b = \sqrt{2}$, $c = \sqrt{6}$

4a) $c = 14$, $b = 7\sqrt{3}$

4b) Both legs have length 15

5b) $a = 2.0838$, $b = 11.8177$

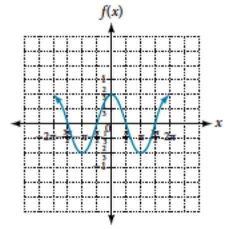
9) 1,060.09 ft

10) 27.372 ft

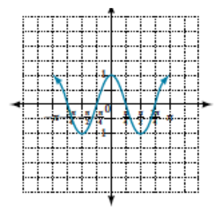
11) 22.6506 ft

Section 6.1

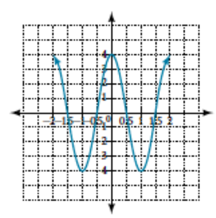
1b) Amplitude: $\dfrac{2}{3}$; period: 2π; midline: $y = 0$; maximum: $y = \dfrac{2}{3}$ occurs at $x = \dfrac{\pi}{2}$; minimum: $y = -\dfrac{2}{3}$ occurs at $x = \pi$; for one period, the graph starts at 0 and ends at 2π. Range; $\left[-\frac{2}{3}, \frac{2}{3}\right]$



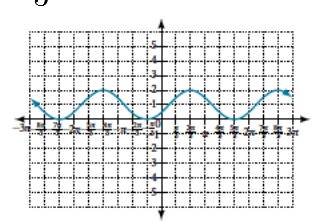
1c) Amplitude: 1; period: π; midline: $y = 0$; maximum: $y = 1$ occurs at $x = \pi$; minimum: $y = -1$ occurs at $x = \dfrac{\pi}{2}$; for one period, the graph starts at 0 and ends at π. Range; $[-1, 1]$



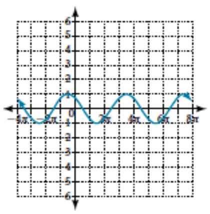
1d) Amplitude: 4; period: 2; midline: $y = 0$; maximum: $y = 4$ occurs at $x = 2$; minimum: $y = -4$ occurs at $x = 1$; for one period, the graph starts at 0 and ends at 2. Range; $[-4, 4]$



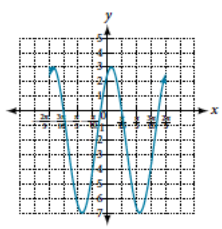
2b) Amplitude: 1; period: 2π; midline: $y = 1$; maximum: $y = 2$ occurs at $x = \dfrac{2\pi}{3}$; minimum: $y = 0$ occurs at $x = \dfrac{5\pi}{3}$; for one period, the graph starts at 0 and ends at 2π; phase shift: $-\dfrac{\pi}{3}$; vertical translation: 1



2d) Amplitude: 1; period: 4π; midline: $y = 0$; maximum: $y = 1$ occurs at $x = \dfrac{11\pi}{3}$; minimum: $y = -1$ occurs at $x = \dfrac{5\pi}{3}$; for one period, the graph starts at 0 and ends at 4π; phase shift: $-\dfrac{10\pi}{3}$; vertical translation: 0



2f) Amplitude: 5; period: $\dfrac{2\pi}{5}$; midline: $y = -2$; maximum: $y = 3$ occurs at $x = 0.08$; minimum: $y = -7$ occurs at $x = 0.71$; for one period, the graph starts at 0 and ends at $\dfrac{2\pi}{5}$. Range; $[-7, 3]$



3a) Amplitude: 2; midline: $y = -3$; period: 4; equation: $f(x) = 2\sin\left(\dfrac{\pi}{2}x\right) - 3$; Domain: $(-\infty, \infty)$; Range: $[-5, -1]$

4b) Amplitude: 2; midline: $y = 3$; period: 5; equation: $f(x) = -2\cos\left(\dfrac{2\pi}{5}x\right) + 3$; Domain: $(-\infty, \infty)$; Range: $[1, 5]$

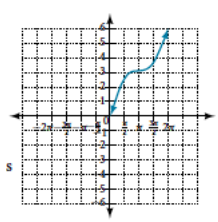
5a) 0, π

5c) 1

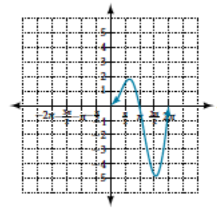
5e) $\dfrac{\pi}{2}$

5g) the origin

6a) The y-coordinates of $y = \sin(x)$ are added to the y-coordinates of the line $y = x$ by stacking the two y-coordinates; thereby winding around the line $y = x$, crossing the line when $y = \sin(x) = 0$ (at $x = 0$, $x = \pi$, $x = 2\pi$,...)



6b) There is no amplitude because the function is not bounded.



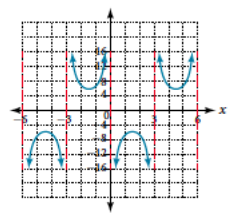
Section 6.2

1b) IV

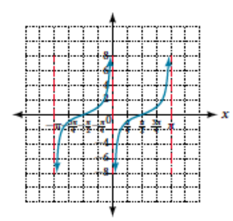
1d) III

2b) Period: 8; horizontal shift: 1; phase shift: -1

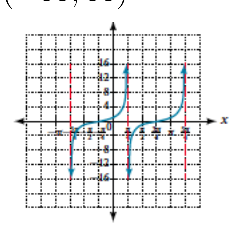
3b) Vertical stretch factor: 6; period: 6; asymptotes: $x = 3k$, where k is an integer; Domain: $\{x|x \neq 3k, \text{where } k \text{ is an integer}\}$; Range: $(-\infty, -6] \cup [6, \infty)$



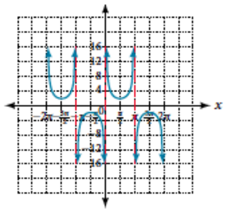
3d) No vertical stretch; period: π; asymptotes: $x = \pi k$, where k is an integer; Domain: $\{x|x \neq \pi k, \text{where } k \text{ is an integer}\}$; Range: $(-\infty, \infty)$



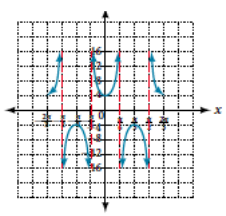
3e) No vertical stretch; period: π; asymptotes: $x = \dfrac{\pi}{4} + \pi k$; Domain: $\{x|x \neq \dfrac{\pi}{4} + \pi k, \text{where } k \text{ is an integer}\}$; Range: $(-\infty, \infty)$



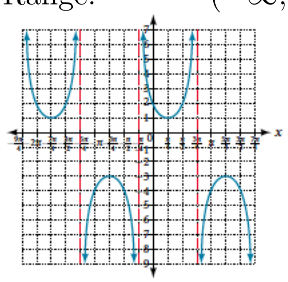
3f) Vertical stretch factor 2; period: 2π; asymptotes: $x = \pi k$, where k is an integer; Domain: $\{x | x \neq \pi k, \text{where } k \text{ is an integer}\}$; Range: $(-\infty, -2] \cup [2, \infty)$



3g) Vertical stretch factor 4; period: $\frac{2\pi}{3}$; asymptotes: $x = \frac{\pi}{6}k$, where k is an odd integer; Domain: $\{x | x \neq \frac{\pi}{6}k, \text{where } k \text{ is an odd integer}\}$; Range: $(-\infty, -4] \cup [4, \infty)$



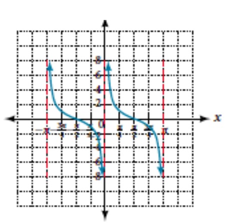
3j) Vertical stretch factor 2; period: 2π; asymptotes: $x = -\frac{\pi}{4} + \pi k$, where k is an integer; Domain: $\{x | x \neq -\frac{\pi}{4} + \pi k, \text{where } k \text{ is an integer}\}$; Range: $(-\infty, -3] \cup [1, \infty)$



4a) $f(x) = \csc(2x)$

4d) $f(x) = \frac{1}{2}\tan(100\pi x)$

5b)



Section 6.3

1) In order for any function to have an inverse, the function must be one-to-one, that is pass the horizontal line test. The regular sine function is not one-to-one unless its domain is restricted in some way. Mathematicians have agreed to restrict the domain of the sine function to the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ so that it is one-to-one and possesses an inverse.

2b) $-\frac{\pi}{6}$

2c) $\frac{3\pi}{4}$

2e) $\frac{\pi}{3}$

3a) 1.98 radians

3b) 0.93 radians

3c) 1.41 radians

4) 0.56 radians

5b) 0.71 radians

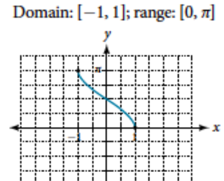
5e) $\frac{4}{5}$

5f) $\frac{5}{13}$

6a) $\frac{x-1}{\sqrt{-x^2+2x}}$

6b) $\frac{\sqrt{x^2-1}}{x}$

8) Domain: $[-1, 1]$; range: $[0, \pi]$



9) 0.40 radians

10) 63.6°

11) 0.41 radians

Section 7.1

1a) $\sin x$

1b) $\sec x$

1c) $\csc t$

1d) $\sec^2\theta$

1e) $\sin^2 x + 1$

3) $-4\sec x\tan x$

Section 7.2

1b) $\dfrac{\sqrt{6}-\sqrt{2}}{4}$

1d) $\sqrt{3}-2$

2a) $-\dfrac{\sqrt{2}}{2}\sin x - \dfrac{\sqrt{2}}{2}\cos x$

2b) $-\dfrac{1}{2}\cos x - \dfrac{\sqrt{3}}{2}\sin x$

3a) $\csc\theta$

3b) $\cot x$

3d) $\tan\left(\dfrac{x}{10}\right)$

5) $\sin(a-b) = \dfrac{4}{5}\left(\dfrac{1}{3}\right) - \dfrac{3}{5}\left(\dfrac{2\sqrt{2}}{3}\right) = \dfrac{4-6\sqrt{2}}{15}$

6b) $\dfrac{\sqrt{2}-\sqrt{6}}{4}$

Section 7.3

2a) $\dfrac{3\sqrt{7}}{32}; \dfrac{31}{32}; \dfrac{3\sqrt{7}}{31}$

2b) $\dfrac{\sqrt{3}}{2}; -\dfrac{1}{2}; -\sqrt{3}$

3) $\cos\theta = -\dfrac{2\sqrt{5}}{5}$, $\sin\theta = \dfrac{\sqrt{5}}{5}$, $\tan\theta = -\dfrac{1}{2}$, $\csc\theta = \sqrt{5}$, $\sec\theta = -\dfrac{\sqrt{5}}{2}$, $\cot\theta = -2$

4a) $2\sin\left(\dfrac{\pi}{2}\right)$

5a) $\dfrac{\sqrt{2-\sqrt{2}}}{2}$

6b) $\dfrac{3\sqrt{13}}{13}; -\dfrac{2\sqrt{13}}{13}; -\dfrac{3}{2}$

6c) $\dfrac{\sqrt{10}}{4}; \dfrac{\sqrt{6}}{4}; \dfrac{\sqrt{15}}{3}$

7a) $\dfrac{120}{169}, -\dfrac{119}{169}, -\dfrac{120}{119}$

7b) $\dfrac{2\sqrt{13}}{13}; \dfrac{3\sqrt{13}}{13}; \dfrac{2}{3}$

8c) $\cos(18x)$

8d) $3\sin(10x)$

Section 7.4

1a) $8(\cos(5x) - \cos(27x))$

2a) $2\cos(5t)\cos t$

2b) $2\cos(7x)$

3a) $\dfrac{1}{4}(1+\sqrt{3})$

3c) $\dfrac{1}{4}(\sqrt{3}-1)$

Section 7.5

2a) $\dfrac{\pi}{6}+\pi k,\ \dfrac{\pi}{3}+\pi k,\ \dfrac{7\pi}{6}+\pi k,\ \dfrac{4\pi}{3}+\pi k,$ where k is any integer

3a) $\dfrac{\pi}{4}, \dfrac{5\pi}{4}$

3b) $\dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{5\pi}{4}, \dfrac{7\pi}{4}$

3d) $\dfrac{\pi}{6}, \dfrac{\pi}{3}, \dfrac{7\pi}{6}, \dfrac{4\pi}{3}$

3f) $\dfrac{\pi}{3}, \pi, \dfrac{5\pi}{3}$

4a) $\pi - \sin^{-1}\left(-\dfrac{1}{4}\right), 2\pi + \sin^{-1}\left(-\dfrac{1}{4}\right), \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$

4b) $\dfrac{\pi}{2}$

4c) π

4d) $\dfrac{7\pi}{6}, \dfrac{11\pi}{6}$

5a) $\cos^{-1}\left(\dfrac{1}{3}(1-\sqrt{7})\right), 2\pi - \cos^{-1}\left(\dfrac{1}{3}(1-\sqrt{7})\right)$

5b) There are no solutions

6a) $\tan^{-1}\left(\dfrac{1}{2}(\sqrt{29}-5)\right), \pi + \tan^{-1}\left(\dfrac{1}{2}(-\sqrt{29}-5)\right), \pi + \tan^{-1}\left(\dfrac{1}{2}(\sqrt{29}-5)\right), 2\pi + \tan^{-1}\left(\dfrac{1}{2}(-\sqrt{29}-5)\right),$

6b) $\dfrac{3\pi}{2}$

6c) $0, \frac{\pi}{3}, \pi, \frac{4\pi}{3}$

6d) There are no solutions.

6e) $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{3}, \frac{5\pi}{3}$

7b) There are no solutions

7d) $0, \frac{2\pi}{3}, \frac{4\pi}{3}$

7f) $\sin^{-1}\left(\frac{3}{5}\right), \frac{\pi}{2}, \pi - \sin^{-1}\left(\frac{3}{5}\right), \frac{3\pi}{2}$

8a) 0.67, 3.81

8c) .72, 5.56

Section 8.1

1a) $\beta = 72°$, $a \approx 12.0$, $b \approx 19.9$

2) $b \approx 3.79$

3a) One triangle, $\alpha = 50.3°$, $\beta = 16.7°$, $a \approx 26.7$

3c) Two triangles, $\gamma \approx 54.3°$, $\beta \approx 90.7°$, $b \approx 20.9$ or $\gamma' \approx 125.7°$, $\beta' \approx 19.3°$, $b' \approx 6.9$

3d) Two triangles, $\gamma \approx 61.3.3°$, $\beta \approx 75.7°$, $b \approx 9.9$ or $\gamma' \approx 118.7°$, $\beta' \approx 18.3°$, $b' \approx 3.2$

3e) No triangle possible

4a) 12.3

5a) 29.7°

6a) 57.1

6b) 430.2

7) 10.1

8) $AD \approx 13.8$

9) 51.4 feet

10) 371 ft

11) 5,936 ft

12) 19,056 sq. ft

Section 8.2

2b) 34.7

3b) 26.9°

4a) $\beta = 45.9°$

5a) 177.56 in^2

6) 18.3

7) 24.0 km

8) 2,371 mi

9) 292.4 mi

10) 468 sq. ft

Section 8.3

1) The point $\left(-3, \frac{\pi}{2}\right)$ has a positive angle but a negative radius and is plotted by moving to an angle of $\frac{\pi}{2}$ and then moving 3 units in the negative direction. This places the point 3 units down the negative y-axis. The point $\left(3, -\frac{\pi}{2}\right)$ has a negative angle and a positive radius and is plotted by first moving to an angle of $-\frac{\pi}{2}$ and then moving 3 units down. The point is also 3 units down the negative y-axis.

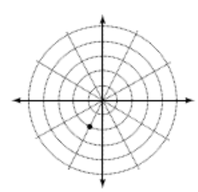
2b) (-5, 0)

2c) $\left(-\frac{3\sqrt{3}}{2}, -\frac{3}{2}\right)$

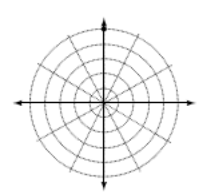
3b) $(5.831, 5.253)$

4b) $\left(3, \frac{3\pi}{4}\right)$

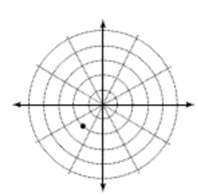
5a)



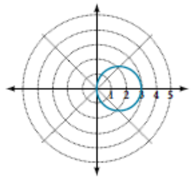
5c)



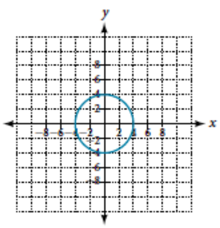
5d)



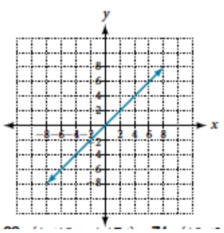
6c) $r = 3\cos\theta$



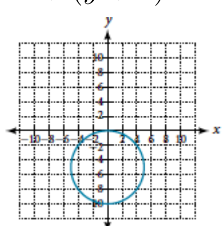
7a) $x^2 + y^2 = 16$



7b) $y = x$



7c) $x^2 + (y+5)^2 = 25$



Section 8.5

1a) $5\sqrt{2}$

1b) $\sqrt{38}$

2a) $8\left(\cos\left(\frac{5\pi}{3}\right) + i\sin\left(\frac{5\pi}{3}\right)\right)$

2c) $2\left(\cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right)\right)$

3a) $\frac{7\sqrt{3}}{2} + \frac{7}{2}i$

3c) $-\frac{3}{2} - \frac{3\sqrt{3}}{2}i$

4a) $4\sqrt{3}\left(\cos\left(198^\circ\right) + i\sin\left(198^\circ\right)\right)$

4b) $\frac{3}{4}\left(\cos\left(180^\circ\right) + i\sin\left(180^\circ\right)\right)$

5a) $7\left(\cos\left(70^\circ\right) + i\sin\left(70^\circ\right)\right)$

5b) $5\left(\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)\right)$

6a) $4 - 4i$

6c) $1,048,576 = 2^{20}$

7a) $3\left(\cos\left(80^\circ\right) + i\sin\left(80^\circ\right)\right), 3\left(\cos\left(200^\circ\right) + i\sin\left(200^\circ\right)\right),$
$3\left(\cos\left(320^\circ\right) + i\sin\left(320^\circ\right)\right)$

7b) $2\sqrt[3]{4}\left(\cos\left(\frac{2\pi}{9}\right) + i\sin\left(\frac{2\pi}{9}\right)\right),$
$2\sqrt[3]{4}\left(\cos\left(\frac{8\pi}{9}\right) + i\sin\left(\frac{8\pi}{9}\right)\right),$
$2\sqrt[3]{4}\left(\cos\left(\frac{14\pi}{9}\right) + i\sin\left(\frac{14\pi}{9}\right)\right)$

7c) $2\left(\cos\left(\frac{7\pi}{12}\right) + i\sin\left(\frac{7\pi}{12}\right)\right),$
$2\left(\cos\left(\frac{15\pi}{12}\right) + i\sin\left(\frac{15\pi}{12}\right)\right),$
$2\left(\cos\left(\frac{23\pi}{12}\right) + i\sin\left(\frac{23\pi}{12}\right)\right)$

Section 8.8

1) $\langle 7, -5\rangle$

2a) Not equal

3) $-7i - 3j$

4b) $-6i - 2j$

5a) $\langle -5, 5\rangle$

5b) $\langle -1, 3\rangle$

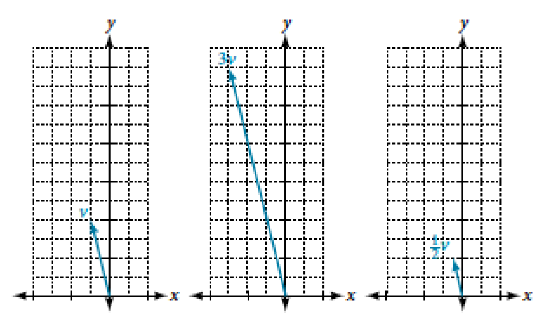
5c) $\langle 0, 5\rangle$

7b) $-\frac{7\sqrt{2}}{10}i + \frac{\sqrt{2}}{10}j$

8a) $|\vec{v}| = 7.810,\ \theta = 5.093$

9) -6

10)



11a)